

## INTERFACIAL HEAT AND MASS TRANSFER UNDER THE TURBULENT MOTION OF FLUIDS\*

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This work represents a further development and generalization of the earlier published theoretical concept of the mechanism of heat and mass transfer across the interface. New findings have been utilized regarding the structure of turbulence near the interface as well as the experience gathered in the application of the above concept to various systems of chemical-engineering interest.

Theoretical analysis has led to a general expression which has been applied to the flow in a tube of circular cross section yielding results agreeing very well with experiments in the whole real range of the Schmidt (Prandtl) number (roughly  $3 \cdot 10^{-3} < Sc(Pr) < 3 \cdot 10^3$ ). Also the conclusions drawn for the region of extremely low and high values of the above parameters are fully compatible with the physical nature of the process.

The progress of our understanding of the hydrodynamics of turbulent flow of fluids, and that near the interface in particular, has necessarily reflected also in the comprehension of the mechanism of transport phenomena across the interface. This has led, apart from the already classic film theory<sup>1</sup>, to the formulation of the Higbie's penetration theory<sup>2</sup>. Although Higbie did not consider the effect of turbulent motion of the fluid on the transport, his idea of an unsteady transport was a novelty which has found use not only in direct application and data processing but has become also an important starting point for further development of theoretical research. The first to consider the effect of turbulence on surface renewal was Kishinevskii<sup>3</sup> who completely disregarded the effect of molecular transport in comparison to the turbulent one. Dackwerts<sup>4</sup> in his modification of the Higbie's theory introduced just like Kishinevskii surface renewal due to turbulence plus unsteady molecular transport into the renewed elements of the fluid similarly as Higbie.

Toor and Marcello in their paper<sup>5</sup> attempted to show that the film and the penetration theory are two limiting cases of the same mechanism. However, unlike the previous authors, they considered the unsteady transport into a layer of finite thickness instead of the semi-infinite one.

Another modification of the penetration theory was proposed by Harriot<sup>6</sup>. This author assumed that the turbulent disturbances generated in the bulk do not reach as far as the interface but only up to a certain distance from the interface bringing about here concentration changes. This distance was then introduced as another parameter. Small values of this parameter lead to the agreement with the penetration theory; large ones to the film theory.

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Beside these theories there exist numerous others which shall not be mentioned in connection with the presented paper.

The main drawback of the proposed theories is that the qualitative concepts are not supported by quantitative descriptions which would comply with the present-date knowledge of the structure of a fluid in turbulent motion in general, and in the proximity of the interface in particular.

An attempt to overcome this drawback has been made already in the earlier published paper<sup>7,8</sup> of which the present communication is an extension and whose results are supplemented and generalized. This has been achieved with the aid of new findings regarding the structure of turbulence near the interface and with the experience gathered in the application of the original concepts to systems of chemical-engineering interest.

### THEORETICAL

The motion of a fluid at high Reynolds numbers is characterized by strong tendency toward isotropy and independence of its statistical characteristics on the character and geometry of the surfaces confining the region of the flow. This inspired Kolmogorov<sup>9,10</sup> to introduce a new notion of locally isotropic turbulence. This notion originates from the fact that at sufficiently high Reynolds numbers turbulent disturbances superimpose on the bulk net flow of the fluid. These disturbances can be characterized by a certain size and velocity and can thus be ascribed a definite value of the Reynolds number. At sufficiently high values of this criterion the fluctuation motion becomes unstable and higher order disturbances superimpose on it. This process goes on until the Reynolds number for the fluctuations of the highest order is so small ( $Re_\lambda = 1$ ) that no higher-order disturbances can appear and all kinetic energy of disturbances in this region dissipates by internal friction into heat.

Because the transport of energy from lower-order disturbances to the higher-order ones is a random process it may be assumed that within a sufficiently small volume of the fluid the elements of turbulence possess from a statistical point of view an approximately isotropic character.

According to Kolmogorov, the distribution function of the relative velocities of fluid elements with respect to an arbitrary element from the given region is a function of the rate of energy dissipation in a unit mass and kinematic viscosity; for sufficiently large distances of these elements only of the former of these two quantities.

The length, velocity and time scale of turbulence is then defined by

$$\lambda = (v^3/\varepsilon)^{1/4}, \quad v = (\varepsilon\nu)^{1/4}, \quad \tau = (\nu/\varepsilon)^{1/2}. \quad (1)-(3)$$

From Eqs (1) and (2) we then have

$$Re_\lambda = (v\lambda)/\nu = 1. \quad (4)$$

The quantity  $\lambda$ , according to Kolmogorov, characterizes "local degree of turbulence".

Even though the above concepts cannot hold in the immediate vicinity of the interface the results of Klebenoff<sup>11</sup> and Laufer<sup>12</sup> indicate that carefully applied assumption of locally isotropic turbulence leads to useful conclusions even in case of the study of wall turbulence.

The effect of the interface on the structure of turbulence in its proximity is different depending on its character. In each case though the interface exercises a certain damping effect giving rise to intensive dissipation of kinetic energy of the turbulent disturbances into heat.

From the viewpoint of the study of transport phenomena the important question is that of whether or not the turbulent disturbances reach as far as the interface. Although present result of various authors differ<sup>13-15</sup>, all seem to indicate that in the case of the flow in a straight pipe with smooth walls there is always a certain layer of the fluid adhering to the interface (even at extremely high values of the Reynolds number) into which the turbulent disturbances do not reach at all or only exceptionally. If the wall is rough, or if the vector of the velocity of bulk motion of the fluid at some points is not parallel to the interface, this layer need not exist at such points at all. In a complete theoretical analysis of transport processes the existence of such a layer where the turbulent disturbances do not reach should be presumed.

Although for the hydrodynamics of fluid flow this layer may be insignificant, its effect on the transport processes will under certain circumstances, *i.e.* at low thermal conductivity or diffusivity, be of extreme importance.

From the hydrodynamic point of view and in accord with practically all hydrodynamic studies we shall therefore assume that the turbulent disturbances advancing from the region of free turbulence toward the interface decay by internal friction of the fluid in region which shall be referred to as the transient region. Adhering immediately to the interface there remains a certain layer of the fluid into which no disturbances reach, this latter layer being termed the laminar layer.

Because in the transient region intensive dissipation of the kinetic energy occurs it will be assumed that only such disturbances can exist in this region which according to Kolmogorov can be assigned the Reynolds number equal unity and whose size equals the thickness of this layer. Further it will be assumed in accord with the previous work<sup>7</sup> that the time interval between two disturbances is given by the time scale of turbulence

$$\tau_p = \lambda_p/v_p = \lambda_p^2/\nu. \quad (5)$$

The region of free turbulence which immediately follows the transient region will be assumed, as far as its hydrodynamics is concerned, to be composed of locally iso-

tropic disturbances in the Kolmogorov's sense. This means that these disturbances obey equations (1)–(4)

$$\lambda_t = (v^3/\bar{\epsilon})^{1/4}, \quad v_t = (\bar{\epsilon}v)^{1/4}, \quad \tau_t = \lambda_t^2/v, \quad (v_t\lambda_t)/v = 1, \quad (6)–(9)$$

where  $\bar{\epsilon}$  designates the mean rate of energy dissipation per unit mass in the system. i.e.:

$$\bar{\epsilon} = \Delta p u_b/h\varrho \quad (10)$$

or after substituting for  $\Delta p$

$$\Delta p = (4f)(h/l)\varrho(u_b^2/2), \quad (11)$$

$$\lambda_t = l/[(2f)^{1/4}(u_b l/v)^{3/4}]. \quad (12)$$

As far as the transport process itself is concerned it is apparent that it is a very complex phenomenon indeed where generally both molecular and random turbulent transport play a role. This causes a sudden change of temperature and/or concentration at a certain position so that molecular transport in all three hydrodynamic regions is essentially unsteady with randomly changing boundary conditions. Moreover, the boundaries between individual regions are not distinct and one region changes continuously into the other. In order that we may quantitatively describe local conditions in region of the interface we have formulated the following physical model:

Due to the turbulent disturbances arriving at time intervals equalling the time scale of turbulence,  $\tau$ , in region equalling in extent the length scale of turbulence,  $\lambda$ , the temperature and/or concentration differences are equalized. During the interval  $\tau$  an unsteady transport of mass and/or heat occurs while the conditions at the boundaries of a given region are independent of time. Temperatures (concentrations) are equalized always to that boundary value which is closer to the temperature (concentration) prevailing in region of free turbulence. For instance, during the transport away from the interface temperatures (concentrations) are equalized to the lower value; during the transport in the opposite direction to the higher value of the two boundary conditions.

This means that for the laminar layer we shall consider the steady molecular transport; for the transient layer the unsteady transport over the period  $\tau_p$  with the boundary conditions independent of time.

The region of free turbulence then represents a series of  $n_t$  linked regions characterized by the length scale  $\lambda_t$  within which the equalization takes place over the period  $\tau_t$ .

Thus in effect we assume that the fluid within a certain region is replaced due to the turbulent transport by the liquid from the turbulent region only partly but the properties of the fluid within this region are homogenized by turbulent mixing.

The above simplified concept is shown schematically in Fig. 1 for the case of the flow away from the interface.

Mathematically we shall further examine only the transport of mass but an analogous procedure applies to heat transfer.

The net flux of mass across the interface is

$$M_i = K_i(c_i - c_1). \quad (13)$$

As we have assumed a steady transport across the laminar layer this whole amount passes also into the transition region where a part of the mass,  $M_{ap}$ , is accumulated and  $(M_i - M_{ap})$  passes into the turbulent region. We can define additional transfer coefficients by

$$M_i = M_p = K_p(c_i - c_p), \quad (14)$$

$$M_{ap} = K_{ap}(c_i - c_p). \quad (15)$$

The mass passing into the turbulent region passes gradually through a series of  $n_t$  hydrodynamically identical layers where always a part of the mass is accumulated and the rest passes into the next layer. Thus we may write

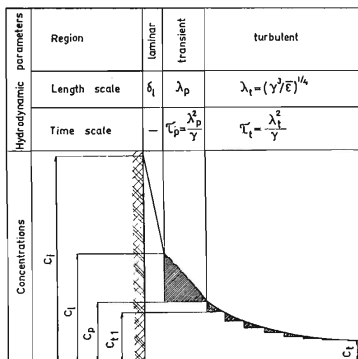


FIG. 1

A Scheme of the Conditions in the Proximity of Interface

Shaded regions represent changes of concentration due to accumulation.

$$M_i - M_{ap} = \sum_1^{n_t} M_{atn} = K_{at}(c_p - c_t), \quad (16)$$

$$M_i = [(1/K_i) + (1/K_p) + (1 - K_{ap}/K_p)(1/K_{at})]^{-1} \cdot (c_i - c_t), \quad (17)$$

where  $c_t$  designates concentration in the turbulent region outside the region of molecular transport. The overall coefficient of mass transfer is then given by

$$K_i = [(1/K_i) + (1/K_p) + (1 - K_{ap}/K_p)(1/K_{at})]^{-1}. \quad (18)$$

The overall mass transfer coefficient in the laminar region is given by

$$K_i = D/\delta_1, \quad (19)$$

and the boundary conditions are  $c(0, t) = c_i$ ,  $c(\delta_1, t) = c_p$ .

In the transition region, where we have assumed an unsteady transport, the mass transfer coefficient is defined as

$$K_p = [\tau_p(c_i - c_p)]^{-1} \int_0^{\tau_p} (-D) (\partial c / \partial x)_{x=\delta_1} dt. \quad (20)$$

The concentration gradient at the laminar/transient region boundary, which appears in Eq. (20), is obtained from the solution of the following partial differential equation

$$(\partial c / \partial t) = D(\partial^2 c / \partial x^2) \quad (21)$$

for  $c(\delta_1, t) = c_i$ ,  $c[(\delta_1 + \lambda_p), t] = c_p$  and  $c(\delta_1 \leq x \leq (\delta_1 + \lambda_p), 0) = c_p$ . Substituting from the familiar form of the solution<sup>16,17</sup> of Eq. (21) for the gradient in Eq. (20) we obtain after integration and substitution for  $\tau_p$  from Eq. (5) the relations

$$(K_p \lambda_p) / D = 1 + (1/3)(v/D) - (2/\pi^2)(v/D) \sum_{n=1}^{\infty} (1/n^2) \exp(-n^2 \pi^2 (v/D)), \quad (22a)$$

or

$$(K_p \lambda_p) / D = (2/\sqrt{\pi})(v/D)^{1/2} [1 + (2/\sqrt{\pi}) \sum_{n=1}^{\infty} \text{ierfc}(n(v/D)^{1/2})]. \quad (22b)$$

The last two expressions are identical but the former converges more rapidly for low values of the parameter  $(v/D)$ ; the latter for larger ones.

At this point it is worth mentioning that the Peclet group should appear in the relations (22a,b) rather than the Schmidt group. However, in our case when  $Re_{\lambda_p} = 1$  the two dimensionless groups take identical values.

For brevity we shall designate the right hand side of Eqs (22a,b) as  $F(v/D)$ .

For sufficiently small values of  $(v/D)$  the third term on the right hand side of Eqs (22a) is negligible and, on the contrary, for sufficiently large values of this parameter the second term in the parentheses on the right hand side of Eqs (22b) may be neglected.

To the accuracy of about 1% we may express  $F(v/D)$  as follows

$$\text{for } v/D \leq 1 \quad F(v/D) = 1 + (1/3)(v/D), \quad (23)$$

$$\text{for } v/D \geq 3 \quad F(v/D) = (2/\sqrt{\pi})(v/D)^{0.5}, \quad (24)$$

$$\text{for } 1 < (v/D) < 3 \quad F(v/D) = 1.32(v/D)^{0.358}. \quad (25)$$

The left hand sides of Eqs (22a,b) represent the Nusselt number related to the length scale of turbulent disturbances.

For the coefficient of mass transfer in the transition region we thus have

$$K_p = (D/\lambda_p) F(v/D). \quad (26)$$

For the rate of accumulation in the transition region we obtain from well-known solutions<sup>16</sup> after substituting for  $t = \tau_p$  from Eq. (5) that

$$M_{ap} = (1/2)(v/\lambda_p) \cdot f(v/D)(c_1 - c_p). \quad (27)$$

Thus

$$K_{ap} = (1/2)(v/\lambda_p) \cdot f(v/D), \quad (28)$$

where

$$f(v/D) = 1 - (8/\pi^2) \sum_{n=0}^{\infty} (1/(2n+1)^2) \exp[-(2n+1)^2 \pi^2/(v/D)]. \quad (29)$$

Finally

$$(K_{ap}/K_p) = (1/2)(v/D) \cdot f(v/D)/F(v/D). \quad (30)$$

For the rate of accumulation in the turbulent layers we can write analogously

$$M_{at} = (1/2)(v/\lambda_t) \cdot f(v/D)(c_p - c_1) \quad (31)$$

and hence

$$K_{at} = (1/2)(v/\lambda_t) \cdot f(v/D). \quad (32)$$

Substituting from Eqs (19), (26) and (32) into Eq. (18) we obtain after recasting into a dimensionless form

$$(K_i l)/D = 1 \cdot F(v/D)/[\lambda_p + \delta_1 \cdot F(v/D) + \lambda_i \Phi(v/D)] \quad (33)$$

where

$$\Phi(v/D) = 2F(v/D)/[(v/D) \cdot f(v/D)] - 1 \quad (34)$$

and  $l$  is a suitable length scale of the system.

For the transport from the turbulent region toward the interface the same relations apply. Instead of accumulation of mass individual regions are stripped of the part of mass that is being transferred away by turbulence.

The courses of the functions  $F(v/D)$  and  $\Phi(v/D)$ , constructed with the aid of the tabulated<sup>16,17</sup> functions  $[1 - f(x)]$  and  $\text{ierfc}(x)$ , are shown in Fig. 2.

From the figure it is apparent that  $F(v/D)$  for low values of the parameter  $(v/D)$  approaches unity while for high values of this parameter the function  $\Phi(v/D)$  vanishes.

From the expression (23) we can conclude that for  $(v/D) \leq 0.03$  we may put  $F(v/D)$  equal to 1, with an error less than 1%. Since in the same interval also the function  $f(v/D)$  approaches unity the function  $\Phi$  reduces to

$$\Phi(v/D) = 2/(v/D) - 1.$$

Hence, for the given region we may write

$$(K_i l/D) = l/[\lambda_p + \delta_1 + [2/(v/D) - 1] \lambda_i]. \quad (35)$$

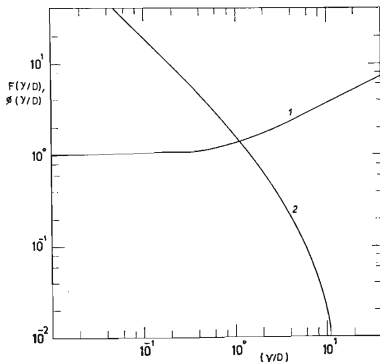


FIG. 2

A Plot of the Functions 1  $F(v/D)$  and 2  $\Phi(v/D)$  versus Schmidt Number



For values of  $(v/D)$  greater than about 100, Eq. (33) reduces to

$$(K_i l/D) = l F(v/D)/[\lambda_p + \delta l F(v/D)]. \quad (36)$$

Eq. (33) applied for the calculations of the overall coefficient of mass transfer,  $K_i$ , requires the knowledge of the physical properties of the system ( $v$ ,  $D$ ), and the hydrodynamic quantities  $\delta_1$  and  $\lambda_p$ .  $\lambda_i$  is given by Eqs (6) and (12) which calls for the coefficients of resistance  $f$  only.

For some simple systems (a tube of circular cross section, a slab) the literature gives relations expressing the thickness of the laminar and transition layer, or the dimensionless distance of the boundaries between individual regions from the solid surface.

These values are to a great extent conventional as may be apparent from the course of  $u^+ = f(x^+)$  which is perfectly continuous in regions where the boundaries between individual layers are usually anticipated.

The thickness of the laminar layer originally given as  $\delta_1^+ = 5$  appears in the light of the new experimental results mentioned in the introduction to be rather  $\delta_1^+ \sim 1$ , and the originally published boundary for the turbulent region  $(\delta_1^+ + \lambda_p^+) \sim 30$  is being decreased by Deyssler<sup>18</sup> to 26.

Thus in order to avoid this uncertainty in the subsequent application of the general expression (33) to the conditions in a smooth tube we shall adopt the following approach: The values  $\delta_1^+$  and  $\lambda_p^+$  shall be calculated from mass or heat transfer data and the obtained values will be examined in the light of the physical meaning as hydrodynamic quantities attached to these parameters.

For a circular pipe thus

$$u^* \delta_1/v = \delta_1^+, \quad u^* \lambda_p/v = \lambda_p^+ \quad (37), (38)$$

and

$$\lambda_i = d/[(2f)^{1/4} \cdot (u_b d/v)^{3/4}]. \quad (39)$$

Substituting from (37), (38) and (39) into Eq. (33) for  $\delta_1$ ,  $\lambda_p$ , and  $\lambda_i$  we obtain after some arrangement

$$\begin{aligned} (K_i l/D) = & (u_b d/v) (f/2)^{1/2} \cdot F(v/D)/[\lambda_i^+ + \delta_p^+ \cdot F(v/D) + \\ & + (1/\sqrt{2}) (u_b d/v)^{1/4} (f/2)^{1/4} \cdot \Phi(v/D)]. \end{aligned} \quad (40)$$

In order to calculate the values  $\delta_1^+$  and  $\lambda_p^+$  we chose for  $Re_b = 10^4$  ( $f = 0.0078$ ) two values of the Schmidt number and found corresponding values of the Sherwood number and the functions  $F(v/D)$  and  $\Phi(v/D)$ .

$$1) \quad (v/D) = 1, \quad (K_i d/D) = 34, \quad F(v/D) = 1.332, \quad \Phi(v/D) = 1.66$$

$$2) \quad (v/D) = 10^2, \quad (K_i d/D) = 226, \quad F(v/D) = 11.263, \quad \Phi(v/D) = 0.$$

The calculation yields:  $\delta_1^+ = 1.0$  and  $\lambda_p^+ = 20$ .

Using these values we then found the course of the Stanton number as a function of the Schmidt number for  $Re_b = 10^4$  and the dependence of the Sherwood number on the Reynolds number for various values of the Schmidt number.

Corresponding dependences are shown in Fig. 3 and 4.

Similarly we could proceed to determine the parameters  $\delta_1^+$  and  $\lambda_p^+$  also for other, more complex systems with the only difference that Eqs (37) and (38) must not be expected to be valid a priori but must be instead first tested experimentally.

If the physical properties of the system significantly vary across the region of interface, which may become significant for the application of the analogue of Eq. (33) to heat transfer, the equation then takes the form

$$(K_i l/D_p) = 1 F(v/D)_p / [\delta_1 (D_p l/D_i) F(v/D)_p + \lambda_p + (v_p f(v/D)_p / v_i f(v/D)_i) \cdot \Phi(v/D)_p \cdot \lambda_i]. \quad (33a)$$

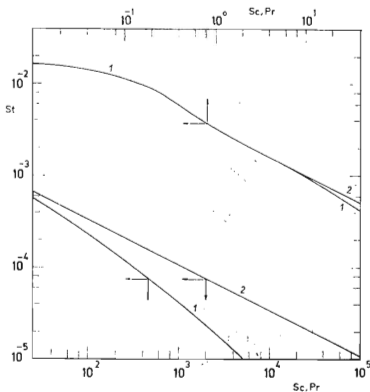


FIG. 3

A Plot of the Function  $St = \psi(v/D)$  for  $Re_b = 10^4$   
Curve 1  $\delta_1^+ = 1$ ; curve 2  $\delta_1^+ = 0$ .

The subscripts indicate the region the conditions of which determine the pertaining physical property.

### DISCUSSION

From the above analysis it follows that the real range of the Schmidt (Prandtl) number is such that the resistance (the inverse of the transfer coefficient) of any of the three hydrodynamic regions may become significant. For low values of the above parameters we may neglect the resistance of the laminar region; at high values the resistance of the turbulent region.

The values of the quantities  $\delta_1^+$  and  $\lambda_p^+$  evaluated for a smooth tube of circular cross section from mass (heat) transfer data are fully compatible with the physical meaning that these quantities are assigned. For instance, Popovich and Hummel<sup>11</sup> using a new photochemic technique found  $\delta_1^+ = 1.6 \pm 0.4$  which is in good agreement with the calculated value. Also the dimensionless distance of the boundary of the region of free turbulence:  $\delta_1^+ + \lambda_p^+ = 21$  agrees with the values published in the literature on hydrodynamics (between 25 and 30). Because in our theory  $\lambda_p^+$  possesses the meaning of a region where intensive dissipation occurs, the lower value comports well with the physical concept. ( $\delta_1^+ + \lambda_p^+$ ) then practically differs

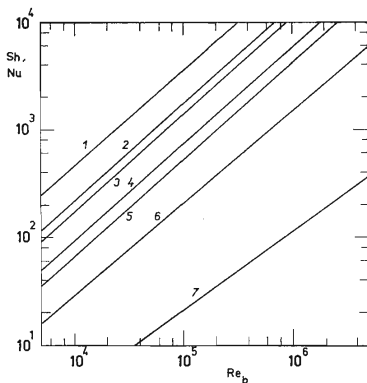


FIG. 4

A Plot of the Function  $Sh = \varphi(Re_b)$  for Various Values of  $Sc(Pr)$   
 1 3000; 2 100; 3 50; 4 10; 5 5; 6 0.7; 7 0.027.

from the value published by Deysler<sup>16</sup> (26) by the correction of the thickness of the laminar layer. Namely, if the thickness of the laminar layer was given as  $\delta_1^+ = 5$  and  $(\delta_1^+ + \lambda_p^+) = 26$  then  $\lambda_p^+ = 21$  which compares favourably with  $\lambda_p^+ = 20$  found by computation from transport data.

For a given value of the Reynolds number at extremely low values of the Schmidt number, the value of the Stanton number approaches a constant indicating asymptotic independence of the mass transfer coefficient on diffusivity (only the turbulent transport becomes important). On the contrary, at extremely high values of Sc the dependence assumes the form  $St = k/Sc$  which agrees with the film theory (only molecular transport important).

Both these conclusions are fully compatible with the physical essence of the process. From Fig. 3 it is apparent that in region  $0.5 \leq Sc$  ( $Pr$ )  $\leq 200$  the often employed dependence  $Sh = f(Re_b, Sc)$  with an exponent of (+0.4) over the Schmidt (Prandtl) number well approximates the course of this dependence.

As far as the dependence  $Sh = f(Re_b)$  at constant Schmidt number is concerned, it may be apparent from Eq. (40) that for the lowest values of the criterion the equation reduces to

$$Sh = k_1 Re_b^{0.75} (f/2)^{0.25}$$

or with the aid of the relation

$$(f/2) = 0.023/Re_b^{0.2}$$

to

$$Sh = k_2 Re_b^{0.7}$$

At high values of the Schmidt number then

$$Sh = k_3 Re_b (f/2)^{0.5}$$

or

$$Sh = k_4 Re_b^{0.9}.$$

In case that  $f = \text{constant}$  (e.g. in rough tubes<sup>19,20</sup> at sufficiently high values of  $Re_b$ :

$$Sh = k_5 Re_b.$$

Thus approximating the dependence of the Sherwood number on the Reynolds number by a power expression the exponents over the Reynolds number for a smooth tube range, depending on the value of the Schmidt number, between 0.7 and 0.9 and for a rough pipe may reach up to 1.

Comparing the computed values of the Stanton number for various values of the Schmidt (Prandtl) number plotted in Fig. 3 with the experimental data from about

ten various sources compiled by Deyssler<sup>18</sup> we found a full agreement in the whole range except for the inherent scatter. In region  $3000 > Sc(Pr) > 200$  the course of the dependence agrees with that presented by Deyssler.

For completeness, Fig. 3 shows also this dependence for the case that  $\delta_1 = 0$ . This line represents the upper limit of the Stanton number which can plausibly exist for a given value of the Schmidt number. Rather closely to this dependence appear the data on dissolution of solids in liquids under mixing<sup>21</sup> and those for the flow through a bed of packing<sup>22</sup>.

From Fig. 3 it is also apparent that the importance of the laminar layer increases with increasing value of the Schmidt (Prandtl) number in spite of the fact that its thickness is small.

The lowest values of the Prandtl number correspond to liquid metals ( $0.003 \leq Pr \leq 0.03$ ).

Although the data in this region exhibit considerable scatter it may be concluded that even in this region the presented theory provides very good results. For this comparison we have furnished the data for  $Pr = 0.027$  corresponding to mercury. Calculated values agree with the experimental data measured by Gilliland and coworkers<sup>23</sup> and Trefethen<sup>24</sup> in the range  $400 < Pe < 2000$ .

Doody and Younger<sup>25</sup> correlated their data for mercury (the presented mean value of  $Pr$  is 0.0243) by the relation

$$Nu = 0.00443 Re_b^{0.69}$$

Our theory with the earlier found values  $\delta_1^+$  and  $\lambda_p^+$  yields

$$Nu = 0.0049 Re_b^{0.71}.$$

## CONCLUSION

The discussion of the results has shown that the application of the general equation (33) to conditions in a smooth tube of circular cross section yields relations that are valid to a high accuracy in the whole real range of the Schmidt (Prandtl) number. Also the limiting values of these parameters yield results compatible with the physical nature of the process. The relation affords explanation for the existence of numerous empirical correlations by the effect of the physical properties or the geometry of the system. The resulting relationship may be applied also to other, more complex systems and offers also a possibility of obtaining information about important hydrodynamic quantities as  $\delta_1^+$  and  $\lambda_p^+$  from the measurement of heat or mass transfer.

## LIST OF SYMBOLS

$c$	concentration
$d$	tube diameter
$D$	diffusivity
$f$	friction coefficient defined by Eq. (11)
$f(v/D)$	function defined by Eq. (30)
$F(v/D)$	function defined by Eq. (23a,b)
$h$	length of tube
$\operatorname{ierfc} x = \int_0^{\infty} \operatorname{erfc} \xi \, d\xi = (1/\sqrt{\pi}) \exp(-x^2) - x \cdot \operatorname{erfc} x$	
$k_1, k_5$	constants
$K$	mass transfer coefficient
$l$	dimension of system
$M$	mass flux
$n$	summation index
$Nu$	Nusselt number
$\Delta p$	pressure drop
$Pe$	Peclet number
$Pr$	Prandtl number
$Re$	Reynolds number
$Sc$	Schmidt number
$Sh$	Sherwood number
$St$	Stanton number
$t$	time
$u_b$	mean velocity
$u^* = u_b \sqrt{f/2}$	friction velocity
$u^+ = u/u^*$	dimensionless velocity
$x$	distance from interface
$x^+ = (u^*x)/\nu$	dimensionless distance
$v$	velocity scale of turbulence
$\epsilon$	rate of energy dissipation per unit mass
$\Phi(v/D)$	function defined by Eq. (35)
$\lambda$	length scale of turbulence
$\nu$	kinematic viscosity
$\rho$	density
$\tau$	time scale of turbulence
$\delta_1$	thickness of laminar layer

## Subscript

a	accumulation
b	related to $u_b$
i	interface
l	laminar region
p	transition region
t	turbulent region

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